TIME-VARYING STATISTICAL COMPLEXITY MEASURES WITH APPLICATION TO EEG ANALYSIS AND SEGMENTATION

P. Celka[†] *and P. Colditz*[§]

[†] Swiss Center for Electronics and Microtechnology, CSEM,
Systems Engineering Division, CH-2007 Neuchâtel, Switzerland
[§]Perinatal Research Centre,
Royal Women's Hospital, Brisbane, Qld 4029, Australia

ABSTRACT

The recently proposed instantaneous statistical dimension is compared to new conditional Rényi entropies. The motivation for introducing these time-varying complexity measures is the analysis of electroencephalograms for which nonstationarity is an inherent property. Experimental data from babies are analyzed using the proposed complexity measures. The instantaneous statistical dimension computation is based on an adaptive autocorrelation eigenspectrum computation known as APEX together with a model selection rule. The conditional Rényi entropies are based on time-frequency representation of the signal. It is shown that: 1) the three time-varying complexity measures account for a *component counting* property, 2) the instantaneous statistical dimension is the most robust to Gaussian white noise.

1. INTRODUCTION

An electroencephalographic signal (EEG) is composed of several types of subsignals, each referring to a particular state of the brain. The complexity of EEG signals originates from the huge number of degrees of freedom of the central nervous system. Neurons and neuronal networks composing the central nervous system can behave in an asynchronous or synchronous manner. Asynchronous activity leads to a more or less continuous background activity, while synchronous activity leads to rhythmical patterns [1]. We know that biological neurons can be modeled by a set of nonlinear differential equations. The coupling of these neurons results in a complex neuronal network that can exhibit different behaviour like spatio-temporal patterns or travelling waves. In both cases, the signal recorded at a particular location often display a *nonstationary* behaviour [1].

Recently, it has been proposed to analyse nonstationary time series with an adaptive principal component neural network [2]. A linear neural network is used to learn and track the eigenvalues $\Lambda(k) = \{\lambda_i(k)\}$, for $i = 1, \ldots, n_s$ and associated eigenvectors $\mathbf{e}_i(k)$ of a time-varying covariance matrix of a time series $x(k)$. A time-varying model selection procedure is then used to select the most relevant eigenvalues and define the *instantaneous statistical dimension* (ISD) $m(k,T)$ (T is a time-averaging horizon). It is shown in this paper that $m(k,T)$ actually count the number of signal components through time. An other time-varying component counting methods is introduced here. It is based on Rényi conditional entropies computed from time-frequency signal representation. Such entropy based time-frequency information measures have been proposed by Williams *et al.* [3] and further studied by Flandrin *et al.* [4]. We propose in this paper to use conditional entropies to compute an *instantaneous* entropy of the signal. We use these measures to analyse EEG signals. It is hoped that these complexity measures will be usefull for feature extraction in EEG signal processing and classification. We finally show that $m(k,T)$ can be used to segment the EEG signal and detect rhytmical activities. The recursive nature of the algorithm that compute $m(k,T)$ makes it suitable for on-line segmentation.

The paper is organized as follows. Section 2 describes the experimental set-up used for recording the electroencephalogram (EEG). Section 3 describes the adaptive method to estimate the eigenvalues $\Lambda(k)$ and defines the ISD. Section 4 presents the conditional Rényi entropies. Section 5 shows some numerical examples and Section 6 experiment these new time-varying complexity measures on EEG signals. Section 7 concludes this paper.

Part of the work has been done at the Signal Processing Research Centre, QUT, Brisbane, Australia. p.celka@qut.edu.au

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Abstract

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2. EXPERIMENTAL SETUP

Five to twenty EEG signals were measured on four babies using electrodes (Ag-AgCl electrodes flushed with conductive gel and adhered by tape) attached to the skin of the newborn. The electrode placement agrees with the American EEG Society standards. Slow baseline fluctuations due to baby movement have been removed by using a 2nd-order high-pass Butterworth filter with a cut-off frequency of 0.1 Hz. These signals were amplified and digitized using either the Amlab© or Medelec© (Oxford Instruments) Software/Hardware environment. The sampling frequency was set to $F_s = 256\,Hz$. EEG signals were then sub-sampled at 40 Hz.

The data acquisitions were performed at the Royal Women Hospital and Royal Children Hospital, Brisbane. Babies were showing electrical seizure activity as labelled by a neurologist from the Neurosciences Department at the Royal Children's Hospital.

3. INSTANTANEOUS STATISTICAL DIMENSION

In the case of nonstationary signals such as the EEG, an adaptive scheme has been proposed to recursively compute the eigenvalue spectrum λ_i, for $i = 1, \ldots, n_s$, of the autocovariance matrix $R_{xx}(k,\tau) = E[x(k)x(k+\tau)]$ of a digital signal $x(k)$ [5]. It uses linear neural networks with Hebbian learning rule to estimate the eigenvectors and eigenvalues. The output $y_i(k)$ of the neural network at the node i at time k is, with input vectors $\mathbf{x}(k) = \mathbf{x}_k = [x(i)x(i+1)\ldots x(i+n_s-1]^T$

$$\begin{aligned} \mathbf{y}_{i-1}(k) &= [\mathbf{p}_1^T(k)\ldots\mathbf{p}_{i-1}^T(k)]^T\mathbf{x}(k) \\ y_i(k) &= \mathbf{p}_i^T(k)\ \mathbf{x}(k) + \mathbf{W}_i^T\ \mathbf{y}_{i-1}(k) \end{aligned} \quad (1)$$

the MA weight vector is $\mathbf{p}_i^T(k) = [p_{i1}\ldots p_{in_s}]$, the AR weight vector is $\mathbf{W}_i^T(k) = [w_{i1}\ldots w_{ii-1}]$, and the output is $\mathbf{y}_i^T(k) = [y_1\ldots y_i]$. From equation (1) we can see that p_{ij} is the weight connecting input $x(k+i)$ to the output y_j, and that w_{ij} is the internal weight that connect the output y_j to the output y_i. Note also that the network architecture is nonsymmetrical at the output layer $\mathbf{W}$. The update rule for the weights $\mathbf{p}_i(k)$ and $\mathbf{W}_i(k)$ is the following

$$\begin{aligned} \Delta\mathbf{p}_i^T(k) &= \beta\ \{\left(y_i(k)\ \mathbf{x}(k) - y_i^2(k)\ \mathbf{p}_i(k)\right)\} \quad (2) \\ \Delta\mathbf{W}_i(k) &= -\beta\ \left(y_i(k)\ \mathbf{y}_{i-1}(k) + y_i^2(k)\ \mathbf{W}_i(k)\right) \end{aligned}$$

The learning rate is controlled by β which can be estimated off-line by $\beta = \min_i\{1/\lambda_i\}$, or on-line by the following recursion formula

$$\beta(k) = \beta(k-1)/(\gamma + \beta(k-1)\ y_i^2(k)) \quad (3)$$

Using equation (3) allows us to have an adaptive step size for each node i in the network. In a stationary environment, the output $y_i^2(k)$ is a stochastic estimate of the eigenvalue λ_i. It is expected that for a nonstationary signal $x(k)$, the output $y_i^2(k)$ will follow the time-variation of the ith eigenvalue of $R_{xx}(k,\tau)$ with some fluctuations around the true eigenvalues due to the stochastic nature of the input signal $x(k)$.

Definition: The *instantaneous statistical dimension* $m(k,T)$ of the time series $x(k)$ at time k for a window length T is given by

$$m(k,T) = \arg\min_{i\in\{1,\ldots,n_s\}} MDL\left(\overline{\mathbf{y}}_{n_s}^2(k), i\right) \quad (4)$$

where $\overline{y}_i^2(k) = (1/T)\sum_{j=k-T/2}^{k+T/2-1} y_i^2(j)$ and $\overline{\mathbf{y}}_{n_s}^2(k) = [\overline{y}_1^2(k),\ldots,\overline{y}_{n_s}^2(k)]^T$. The minimum description length [2] $MDL(\Lambda(k), i)$ depends on the values $\overline{\mathbf{y}}_{n_s}^2(k)$ at the time instant k. The MDL function also depends on the number of samples $N_T = max\{1/1-\gamma, T\}$ taken into account in the adaptive algorithm.

Note that we average the outpout power $\overline{y}_i^2(k)$ over a symmetric time interval about k. The time averaged instantaneous embedding dimension $m(k,T)$ depends on the time location k and also on the window size T.

While it has been shown that there is no simple relation between $m(k,T)$ and the dimension of the underlying dynamical system, $m(k,T)$ provides us with the following informations: 1) if $m(k,T) \geq n_w$, the signal contains a nonstochastic activity of *high* dimension. This could reflects asynchronous behaviour of the brain neurons. If $1 < m(k,T) < n_w$, the signal contains a nonstochastic activity of *low* dimension. This is typical from synchronous activity of the neurons.

4. CONDITIONAL RÉNYI ENTROPIES

An interesting attempt to measure the information content in time-varying signal $x(t)$ has been made in [3, 4]. They introduced the Rényi entropies $H_R^\alpha(C(t,f))$ of time-frequency distributions (TFD) of Cohen's class $C(t,f)$ assuming a probability density function (pdf) interpretation of $C(t,f)$. They show that $H_R^{(\alpha)}(C(t,f))$ actually count the number of component in a signal. Unfortunately, we loose the time location aspect when we compute the scalar value $H_R^{(\alpha)}(C(t,f))$. We thus define the conditional Rényi entropies of a normalized TFD which allows to introduce instantaneous Rényi entropies and thus a time-varying component counting property. Let us first define the conditional pdf

$C(f|t) = C(t,f)/C(t)$ where $C(t) = \int C(t,f)\ df$. The conditional Rényi entropies for $\alpha > 0$ is

$$H_R^{(\alpha)}(C(f|t)) = \frac{1}{1-\alpha}\ \log_2 \int C^{\alpha}(f|t)\ df \qquad (5)$$

Note that $H_R^{(\alpha\to 1)}(C(f|t)) \to H_S(C(f|t))$ which is, by definition, the conditional Shannon entropy. We can recover the entropy $H_R^{(\alpha)}(C(t,f))$ by

$$H_R^{(\alpha)}(C(t,f)) = \frac{1}{1-\alpha}\ \log_2 \int 2^{(1-\alpha)H_R^{(\alpha)}C(f|t)} C^{\alpha}(t)\ df \qquad (6)$$

The Rényi entropies $H_R^{(\alpha)}(C(t,f))$ and $H_R^{(\alpha)}(C(f|t))$ have units of bits. The Shannon entropy can be computed $H_S(C(t,f)) = H_R^{(\alpha\to 1)}(C(t,f))$. The entropies $H_R^{(\alpha)}(C(f|t))$ depends on the time which makes them suitable for defining time-varying complexity measures. Suitable values for alpha are small odd integer [4] such as $\alpha = 3$.

5. NUMERICAL EXAMPLES

In order to validate our two time-varying complexity measure and assess their time varying component counting properties, we construct the following signal $x(t) = x_1(t)+x_2(t)+n(t)$ where $x_i(t) = sin(\omega_i(t)t)e^{-(t-t_i)^2/2\gamma_i^2}$, $\omega_i(t) = 2\pi(f_i^{(o)} + \alpha_i t)$, and $n(t)$ is a Gaussian White noise. In the following we have $t_1 = 10s$, $t_2 = 12s$, $\gamma_1 = 6s$, $\gamma_2 = 3s$, $f_1^{(o)} = 2Hz$, $f_2^{(o)} = 3Hz$, $\alpha_1 = 0.15s^{-2}$, $\alpha_2 = 0.2s^{-2}$. The signal contains 1 component up to about $10s$ and then one more component is added. The TFD used in these examples is the spectrogram, and Figure 1 shows the spectrogram of the signal $x(t)$ together with the two components.

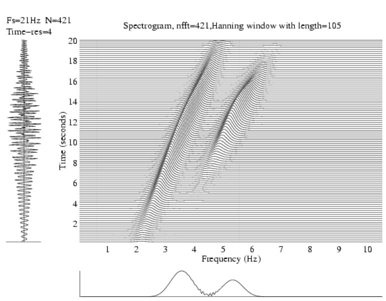

Figure 1: Spectrogram of $x(t)$ for $SNR = 100dB$. Time signal and power spectral density functions are displayed on left and bottom of the figure. Sampling frequency, window type and window length are aslo displayed on the figure.

Three examples are displayed in Figure 2, using $SNR = 100, 20, 10dB$. We observe that, for $SNR = 100dB$, $H_R^{(\alpha)}(C(f|t))$ increases by 1 bit as the signal move from 1 to 2 components. We also observe the border effects that are due to numerical implementation of the spectrogram. The slow transition around $10s$ is due to the time-spreading effect in using the spectrogram.

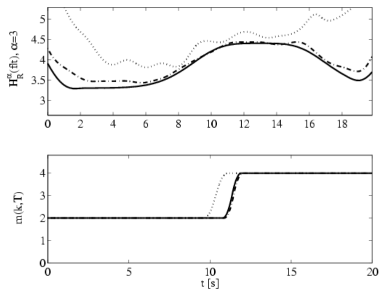

Figure 2: Conditional Rényi entropies and ISD for $SNR = 100dB$ (solid lines), $SNR = 20dB$ (dotted-dashed lines), and $SNR = 10dB$ (dotted lines)

Figure 2 shows the great robustness against noise (at least Gaussian white noise) for the ISD. Moreover, the transition region around the $10s$ is very short. The algorithm show very fast convergence for all SNR. The component counting property is illustrated in these examples.

One crutial distinction between entropy- and ISD-based complexity measures is the influence of noise. As $H_R^{(\alpha)}(C(f|t))$ is computed from a TFD of the noisy signal, this quantity will be highly affected by the later because the noise spread its power in frequency by essence. On the other hand, the ISD measure tend to dissociate the deterministic from the stochastic components and provide a complexity measure of, essentially, the derterministic part. This will be further illustrated in the next section.

6. EXPERIMENTAL RESULTS

The two major biosignal processing applications are detection (or segmentation) and feature extraction for classification and diagnosis. The goal of designing these time-varying complexity measures follows exactely these two directions when dealing with nonstationary signals. First the set of values of the entropies and the ISD can be used in their own as features of the EEG signal. Second, using thresholding methods we can use them to segment the EEG. All the figures 3 to 5 shows the ISD (dotted-dashed lines) and $H_R^{(\alpha=3)}(C(f|t))$ (solid lines). Figure 3 shows a similar decreasing behavior of both complexity measures.

Figure 4 shows an almost constant conditional entropy, while ISD display clearely large fluctuations when EEG rhytms take place. Still, carefull observation of

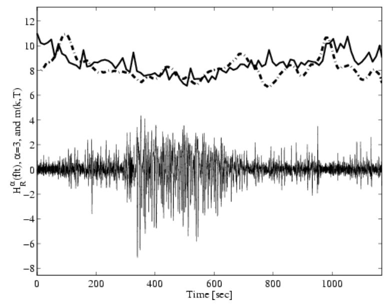

Figure 3: ISD and $H_R^{(\alpha=3)}(C(f|t))$ of one EEG channel

both measures reveal some positive correlations. In this figure, ISD increased during the main ictal segments.

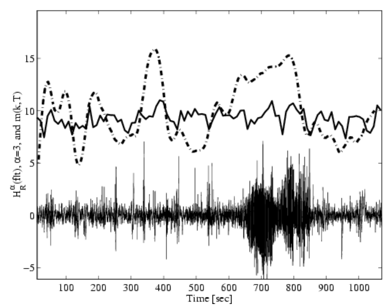

Figure 4: ISD and $H_R^{(\alpha=3)}(C(f|t))$ of one EEG channel

Figure 5 displays a quite constant $H_R^{(\alpha=3)}(C(f|t))$, while ISD varies considerably when EEG rhytmical activity take place. In this figure, the ISD tends to decrease during the ictal segments while increases in the inter-ictal ones. This does not contradict the two previous results, but simply emphasize the fact that ISD provide a complexity measure of the deterministic part of the signals.

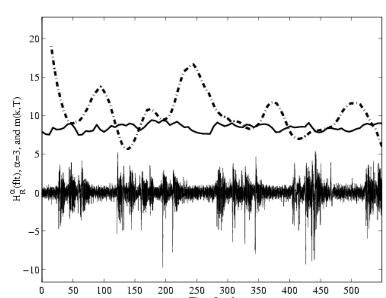

Figure 5: ISD and $H_R^{(\alpha=3)}(C(f|t))$ of one EEG channel

7. CONCLUSIONS

Physiological measurement such as EEG display some nonstationary behaviour. In order to characterize such nonstationary mixture of deterministic and stochastic behaviour, we have developed two time-varying complexity measures. The first one is based on eigenvalue tracking and the second one derived from the generalised conditional Rényi entropies. While sharing the same component counting properties, the ISD revealed to be more robust in noisy measurements such as EEG signals. It is expected that these new concepts will leads to new and interesting tools for the analysis of nonstationary time series. Especially in the case of noisy measurements, MDL selection rule has been shown to perform well in extracting relevant features in the signal. Experimental analysis of EEG signals has been performed and the time-varying complexity analysis reveals some interesting insight into the brain function.

8. ACKNOWLEDGMENT

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